Find $\binom{42}{7}$.

$$\frac{42!}{7!\,35!} = \frac{\cancel{42} \times 41 \times \cancel{40} \times 39 \times 38 \times 37 \times \cancel{36}^{12}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$ (1) (1)

SCORE: _____ / 4 PTS

$= 41 \times 39 \times 38 \times 37 \times 12$ (1) ← ANY PRODUCT WHICH GIVES CORRECT FINAL ANSWER + DOESN'T INVOLVE DIVISION OR !

$= 26{,}978{,}328$ (1)

Find the value of $\sum_{p=1}^{\infty} \frac{16}{3^{2p}}$. **HINT: Write out several terms of the series first.**

SCORE: _____ / 4 PTS

$$= \frac{16}{3^2} + \frac{16}{3^4} + \frac{16}{3^6} + \cdots = \frac{\boxed{\frac{16}{9}}}{\boxed{1 - \frac{1}{9}}} = \frac{\frac{16}{9}}{\frac{8}{9}} = \boxed{2}$$

$\times \frac{1}{9}$ $\times \frac{1}{9}$

1 2 1

An employer ran a year-long charity campaign. During the seventh week, $716 was donated. **SCORE: _____ / 5 PTS**

[a] If the amount donated every week was $24 higher than the amount donated the previous week, how much was donated over the entire year (52 weeks) ?

$a_7 = a_1 + 6d$

$716 = a_1 + 6(24)$ 1

$a_1 = 572$ $\frac{1}{2}$

$S_{52} = \frac{52}{2}(2(572) + 51(24))$ 1

$= 61{,}568$ $\frac{1}{2}$

[b] If the amount donated every week was 3.2% higher than the amount donated the previous week, how much was donated the first week ? **Round your answer to the nearest cent.**

$a_7 = a_1 r^6$

$716 = a_1(1.032)^6$ 1

$a_1 = 592.70$ 1

Using mathematical induction, prove that $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}$ **SCORE: _____ / 12 PTS**

for all positive integers n.

BASIS STEP: $1^2 = 1 = (-1)^2 \frac{1(2)}{2}$

INDUCTIVE STEP:

ASSUME $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k+1} k^2 = (-1)^{k+1} \frac{k(k+1)}{2}$

FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k+2}(k+1)^2$$

$$= 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k+1} k^2 + (-1)^{k+2}(k+1)^2$$

$$= (-1)^{k+1} \frac{k(k+1)}{2} + (-1)^{k+2}(k+1)^2$$

$$= (-1)^{k+1}(k+1)\left[\frac{k}{2} + (-1)(k+1)\right]$$

$$= (-1)^{k+1}(k+1)\left(\frac{k}{2} - k - 1\right)$$

$$= (-1)^{k+1} \frac{k+1}{2}(k - 2k - 2)$$

$$= (-1)^{k+1} \frac{k+1}{2}(-k-2)$$

$$= (-1)^{k+1} \frac{k+1}{2}\left[-(k+2)\right]$$

$$= (-1)^{k+2} \frac{(k+1)(k+2)}{2}$$

BY M.I.

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n+1} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}$$

FOR ALL POSITIVE INTEGERS

Use the entries of Pascal's Triangle to expand and simplify $(7n^3 - 2n^5)^4$. 1 4 6 4 1 **SCORE: _____ / 5 PTS**

<u>You must show the intermediate step in the expansion to get full credit.</u>

$$(7n^3)^4 + 4(7n^3)^3(-2n^5) + 6(7n^3)^2(-2n^5)^2 + 4(7n^3)(-2n^5)^3 + (-2n^5)^4$$

$$= 2401n^{12} - 2744n^{14} + 1176n^{16} - 224n^{18} + 16n^{20}$$

$\frac{1}{2}$ POINT EACH